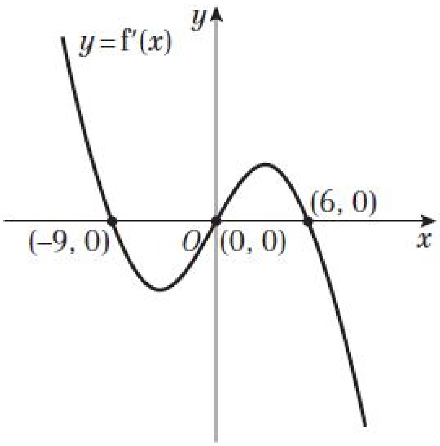
Differentiation 12J

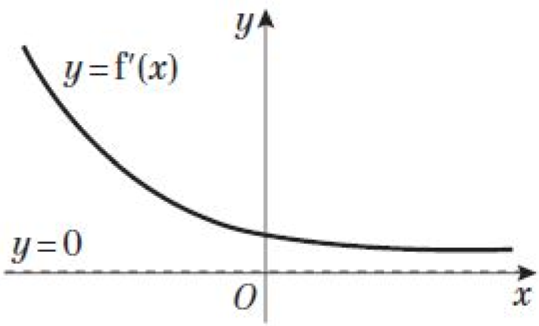
1 a

x	$y = f(x)$	$y = f'(x)$
$x < -9$	Positive gradient	Above x-axis
$x = -9$	Maximum	Cuts x-axis
$-9 < x < 0$	Negative gradient	Below x-axis
$x = 0$	Minimum	Cuts x-axis
$0 < x < 6$	Positive gradient	Above x-axis
$x = 6$	Maximum	Cuts x-axis
$x > 6$	Negative gradient	Below x-axis



b

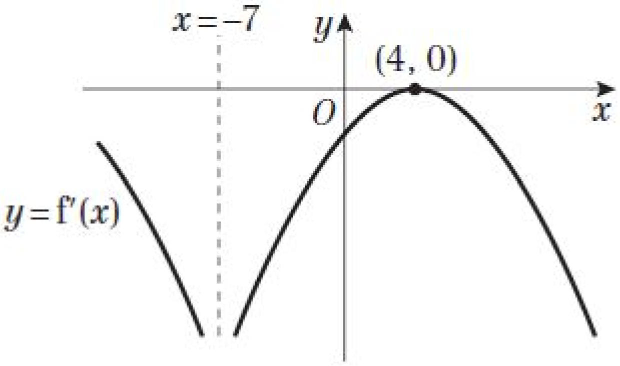
x	$y = f(x)$	$y = f'(x)$
All values of x	Positive gradient	Above x-axis with asymptote at $y = 0$



c

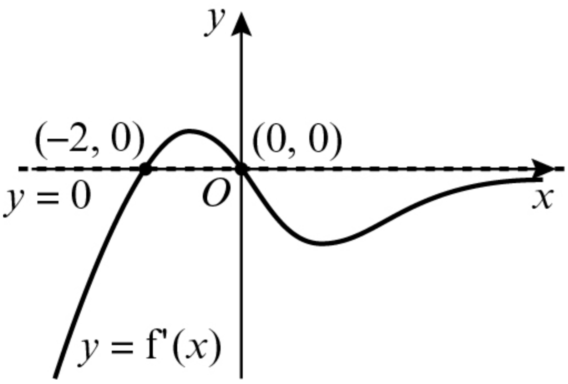
x	$y = f(x)$	$y = f'(x)$
$x < -7$	Negative gradient	Below x-axis with asymptote at $x = -7$
$-7 < x < 4$	Negative gradient	Below x-axis
$x = 4$	Point of inflection	Touches x-axis
$x > 4$	Negative gradient	Below x-axis

1 c



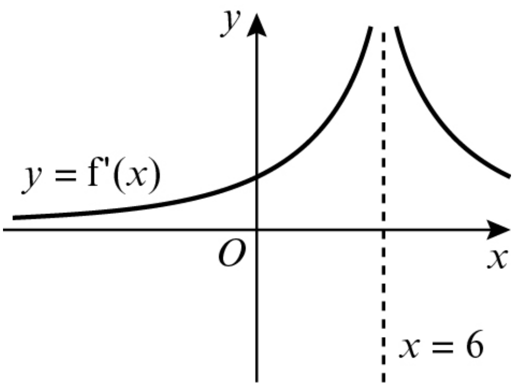
d

x	$y = f(x)$	$y = f'(x)$
$x < -2$	Negative gradient	Below x-axis
$x = -2$	Minimum	Cuts x-axis
$-2 < x < 0$	Positive gradient	Above x-axis
$x = 0$	Maximum	Cuts x-axis
$x > 4$	Negative gradient	Below x-axis with asymptote at $y = 0$



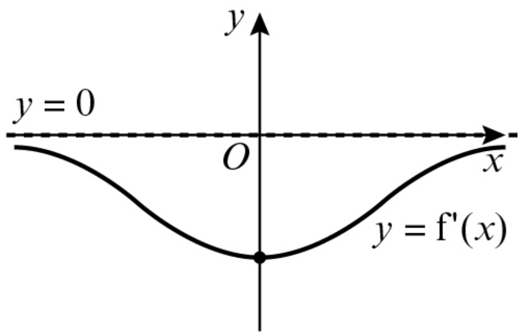
e

x	$y = f(x)$	$y = f'(x)$
$x < 6$	Positive gradient	Above x-axis with asymptote at $x = 6$
$x > 6$	Positive gradient	Above x-axis with asymptote at $x = 6$



1 f

x	$y = \text{f}(x)$	$y = \text{f}'(x)$
$x < 0$	Negative gradient	Below x-axis with asymptote at $y = 0$
$x > 0$	Negative gradient	Below x-axis with asymptote at $y = 0$



2 a $y = \text{f}(x) = (x + 1)(x - 4)^2 = x^3 - 7x^2 + 8x + 16$

When $y = 0$, $x = -1$ or $x = 4$

To find stationary points, $\frac{\text{d}y}{\text{d}x} = 0$:

$$\frac{\text{d}y}{\text{d}x} = 3x^2 - 14x + 8$$

$(3x - 2)(x - 4) = 0$

$x = \frac{2}{3}$ or $x = 4$

When $x = \frac{2}{3}$, $y = \left(\frac{2}{3}+1\right)\left(\frac{2}{3}-4\right)^2 = \frac{500}{27}$

When $x = 4$, $y = (4 + 1)(4 - 4)^2 = 0$

So $\left(\frac{2}{3}, \frac{500}{27}\right)$ and (4, 0) are stationary points.

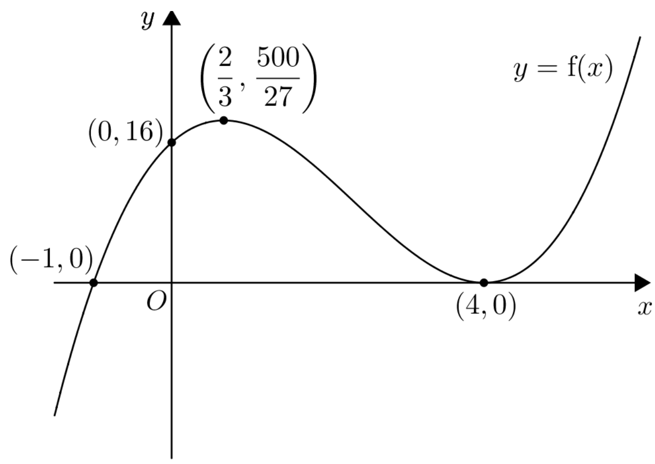
$$\frac{\text{d}^2y}{\text{d}x^2} = 6x - 14$$

When $x = \frac{2}{3}$, $\frac{\text{d}^2y}{\text{d}x^2} = 6\left(\frac{2}{3}\right) - 14 = -10 < 0$

So $\left(\frac{2}{3}, \frac{500}{27}\right)$ is a maximum.

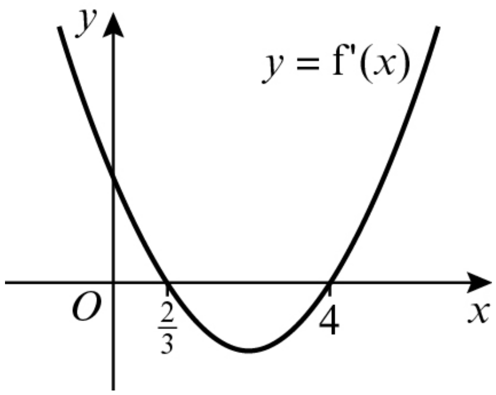
When $x = 4$, $\frac{\text{d}^2y}{\text{d}x^2} = 6(4) - 14 = 10 > 0$

So (4, 0) is a minimum.



2 b

x	$y = \text{f}(x)$	$y = \text{f}'(x)$
$x < \frac{2}{3}$	Positive gradient	Above x-axis
$x = \frac{2}{3}$	Maximum	Cuts x-axis
$\frac{2}{3} < x < 4$	Negative gradient	Below x-axis
$x = 4$	Minimum	Cuts x-axis
$x > 4$	Positive gradient	Above x-axis



c $\text{f}(x) = (x + 1)(x - 4)^2 = x^3 - 7x^2 + 8x + 16$
$\text{f}'(x) = 3x^2 - 14x + 8$
$= (3x - 2)(x - 4)$

d $\text{f}'(x) = 3x^2 - 14x + 8$
$(3x - 2)(x - 4) = 0$
$x = \dfrac{2}{3}$ or $x = 4$
When $x = 0$, $\text{f}'(x) = 8$
The points where the gradient function cuts the axes are $\left(\frac{2}{3}, 0\right)$, (4, 0) and (0, 8).